A Quasi-Static Modification of TLM at Knife Edge and 90° Wedge Singularities

Lucia Cascio, Giampaolo Tardioli, Tullio Rozzi, and Wolfgang J. R. Hoefer

***Abstract*—A common drawback of numerical techniques such as transmission line method (TLM) and finite-difference time-domain method (FDTD) resides in the difficulty to accurately describe the electromagnetic field in structures with singularities. In this paper a local modification of the two-dimensional (2-D) TLM algorithm for the nodes surrounding a knife edge and a 90° wedge is proposed. A quasi-static approximation of the field is used to derive an equivalent circuit of the edge. The proposed theory is then extended to the characterization of infinitely thin septa, the vertex of which is located anywhere between the nodes of the TLM mesh. The proposed corner correction is compared with the uncorrected TLM results and with data available in the literature, revealing a marked enhancement in the accuracy and convergence of the results.**

I. Introduction

THE transmission line method (TLM) [1] is widely regarded as an efficient and flexible technique for the analysis of a large class of electromagnetic problems. One of the main limitations of this and other numerical techniques is that the spatial discretization fails to accurately describe the singularities of the electromagnetic field, which occur for example close to sharp edges.

Unless a very fine discretization is used, the singular behavior around the corner is poorly represented and the frequency domain characteristics of the structure will typically be shifted. This error is very often unacceptable when we are dealing with narrowband structures such as filters.

The accuracy of the discretized model can be improved by introducing a better description of the field singularity, through local modification of the algorithm.

An approach based on the local modification of the standard TLM method to account for the energy stored around the edge has been proposed in [2]. The nodes surrounding the corner are loaded with stubs with optimized characteristics.

In this paper a new approach based on the quasi-static approximation of the Green's functions for an infinite conductive wedge is proposed. The field distribution around a corner is represented in terms of an equivalent circuit which can be implemented easily and efficiently in TLM. The accuracy of the proposed method is compared to that of the standard two-dimensional (2-D) TLM algorithm by means of test structures for which the results are also available in the literature.

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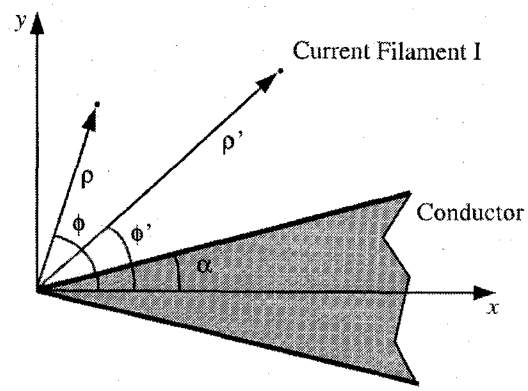


Fig. 1. Conducting wedge.

II. Theory

Consider a current filament adjacent and parallel to a conducting wedge (Fig. 1) where (ρ', ϕ') indicates the source point, and (ρ, ϕ) the field point. The excitation is an impulsive current of strength I.

In this 2-D problem the electric field component E_z in cylindrical coordinates can be expressed as a series of trigonometric and Bessel functions [3]

$$E_z(\rho, \phi) = G(\rho, \phi; \rho', \phi')I \tag{1}$$

$$G(\rho, \phi; \rho', \phi') = -\frac{\omega\mu\pi}{2(\pi-\alpha)} \cdot \begin{cases} \sum_{n=1}^{\infty} H_\nu^{(2)}(k\rho')J_\nu(k\rho)\sin(\nu(\phi-\alpha)) \\ \cdot\sin(\nu(\phi'-\alpha)) \qquad \rho<\rho' \\ \sum_{n=1}^{\infty} H_\nu^{(2)}(k\rho)J_\nu(k\rho')\sin(\nu(\phi-\alpha)) \\ \cdot\sin(\nu(\phi'-\alpha)) \qquad \rho>\rho' \end{cases}$$

$$\text{where} \quad \nu = \frac{n\pi}{2(\pi-\alpha)}. \tag{2}$$

The expressions in (2) present a complex frequency dependence, the variable k being a function of ω. Using approximations for the Bessel and Hankel functions for small values of the argument [4], (2) can be considerably simplified leading

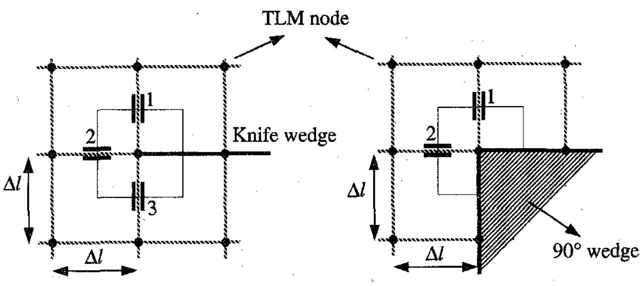


Fig. 2. Knife edge and 90° wedge position in the TLM mesh.

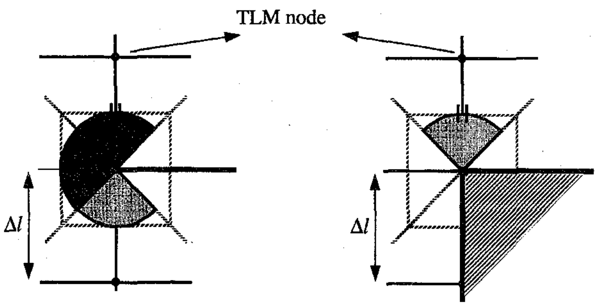


Fig. 3. Domains of integration for the determination of Z.

to a quasi-static solution given by [5]

$$G(\rho, \phi; \rho', \phi') = -j\frac{\omega\mu}{2(\pi-\alpha)} \cdot \begin{cases} \sum_{n=1}^{\infty} \frac{1}{\nu}\left(\frac{\rho}{\rho'}\right)^{\nu} \sin\nu(\phi-\alpha)\sin\nu(\phi'-\alpha) \\ \rho<\rho' \\ \sum_{n=1}^{\infty} \frac{1}{\nu}\left(\frac{\rho'}{\rho}\right)^{\nu} \sin\nu(\phi-\alpha)\sin\nu(\phi'-\alpha). \\ \rho>\rho'. \end{cases} \quad (3)$$

III. Application to the TLM Mesh

The quasi-static expression for the electric field described in (3) represents the basis for the determination of an equivalent circuit describing the field around the edge.

In order to reduce the number of ports surrounding the edge, the conducting boundaries have been placed on the nodes of the TLM mesh. In the case of a knife edge ($\alpha = 0°$) a three-port equivalent circuit is required to characterize the edge behavior, while in the case of a 90° wedge ($\alpha = 45°$) a two-port equivalent circuit is sufficient (Fig. 2).

Since the voltages and currents at the ports are related to the electric field E_z and to the current density J_z (4), we can describe the equivalent circuit by a Z matrix representation [6], [7]. For each port of the circuit we may define

$$V_i \to E_z(\rho_i, \phi_i)$$
$$I_i \to -J_z(\rho_i, \phi_i) = -\frac{I(\rho_i, \phi_i)}{2\pi\rho_i} = -\frac{I(\rho_i, \phi_i)}{\pi\Delta l};$$
$$\rho_i = \tfrac{\Delta l}{2}; \quad = i\tfrac{\pi}{2} \quad (4)$$

where i indicates the number of the port in the circuit.

Consequently, the impedance elements are defined as

$$Z_{ij} = -\pi\Delta l G(\rho_i, \phi_i; \rho_j', \phi_j') = \overline{G}(\rho_i, \phi_i; \rho_j', \phi_j'). \quad (5)$$

Due to the linear dependence of the Green's function on the frequency (3), the equivalent circuit is composed by inductive elements.

A more general definition for the impedances is given by

$$Z_{ij} = \frac{1}{W_i W_j}\int_{W_i}\int_{W_j} \overline{G}(s; s')\, ds\, ds' \quad (6)$$

where $\overline{G}(s; s')$ represents the Green's function determined in (5), and W_i, W_j are the domains of integration for the source variables and for the field variables. The adopted domains of integration are 90° circular sectors, which account for the energy stored around the corner. In Fig. 3 are reported the domains of integration for both the knife edge and the 90° corner. Since we place the boundary on the TLM nodes, (Fig. 2) the number of TLM ports connected to the corner edge is, three for the knife edge, and two for the 90° corner. The correspondent Z matrices are therefore, respectively, of dimension 3×3 and 2×2. Due to the reciprocity of the Green's function and to the geometrical symmetry of the problem there are only four distinct elements z_{ij} for the knife edge equivalent circuit, and only two for the 90° wedge

$$[Z] = j\frac{\omega\mu}{\pi}\begin{bmatrix} z_{11} & z_{12} & z_{13} \\ z_{12} & z_{22} & z_{12} \\ z_{13} & z_{12} & z_{11} \end{bmatrix} = j\omega[\overline{Z}]$$
$$[Z] = j\frac{\omega\mu}{\pi}\begin{bmatrix} z_{11} & z_{12} \\ z_{12} & z_{22} \end{bmatrix} = j\omega[\overline{Z}]. \quad (7)$$

The obtained impedances are, respectively

$$z_{ij} = \left(\frac{16}{\pi}\right)^2 \sum_{n=1}^{\infty} \frac{1}{n^3(4+n)} \sin\left(\frac{n}{2}\varphi_i\right) \cdot \sin\left(\frac{n}{2}\varphi_j\right)\left[\sin\left(\frac{n\pi}{8}\right)\right]^2 \quad i, j = 1, 2, 3 \quad (8)$$

$$z_{ij} = \frac{108}{\pi^2} \sum_{n=1}^{\infty} \frac{1}{n^3(3+n)} \sin\left(\frac{2n}{3}\varphi_i\right) \cdot \sin\left(\frac{2n}{3}\varphi_j\right)\left[\sin\left(\frac{n\pi}{6}\right)\right]^2 \quad i, j = 1, 2. \quad (9)$$

IV. Discretization Process

In order to realize the equivalent circuit in the TLM mesh, we need to determine the relation between the incident and reflected voltages at the ports as a function of the Z matrix elements (6). Due to the quasi-static approximation, the voltages at the ports of the equivalent circuits depend only linearly on the frequency

$$\overline{V} = j\omega[\overline{Z}]\overline{I}$$

$$\overline{V} = \begin{bmatrix} V_1 \\ \vdots \\ V_{n-\text{ports}} \end{bmatrix}; \quad \overline{I} = \begin{bmatrix} I_1 \\ \vdots \\ I_{n-\text{ports}} \end{bmatrix}. \quad (10)$$

The vectors $\overline{V}$ and $\overline{I}$ can be expressed in terms of the incident and reflected TLM voltages at the edge, $\overline{V}^{\,i}$ and $\overline{V}^{\,r}$, according to the relation

$$\overline{V} = \overline{V}^i + \overline{V}^r \qquad \overline{I} = Y_0(\overline{V}^i - \overline{V}^r) \tag{11}$$

where Y_0 is the TLM link line admittance.

The frequency dependance $j\omega$ is discretized using a bilinear transformation [8]; this scheme guarantees the stability of the discretized model. The frequency distortion introduced is irrelevant for frequencies propagating with low dispersion error in the TLM mesh ($\lambda \geq 10 \cdot \Delta l$). Hence

$$j\omega \approx j\frac{2}{\Delta t}\tan\left(\frac{\omega\Delta t}{2}\right) = \frac{2}{\Delta t}\left(\frac{1-e^{-j\omega\Delta t}}{1+e^{-j\omega\Delta t}}\right);$$

$$V_k e^{-j\omega\Delta t} = V_{k-1}. \tag{12}$$

Substituting relations (11) and (12) in (10), we obtain a recursive formulation (13) characterizing the corner condition in the TLM process

$$\overline{V}^r_k = \left(\frac{2}{\Delta t}Y_0[\overline{Z}] - [I]\right)\left(\frac{2}{\Delta t}Y_0[\overline{Z}] + [I]\right)^{-1} \cdot (\overline{V}^i_k + \overline{V}^r_{k-1}) - \overline{V}^i_{k-1}. \tag{13}$$

In this expression Y_0 is the TLM link line admittance, and $\overline{V}^r_k$ and $\overline{V}^i_k$ are the vectors of the voltages incident and reflected at the terminals of the equivalent circuit at the time step k.

V. Generalization to Knife Edge Septa of Arbitrary Length

So far, we have restricted our study to metallic wedges in which the vertex coincided with a TLM node. A useful extension of the theory described above is the characterization of infinitely thin septa, the vertex of which is placed anywhere between the nodes of TLM mesh (Fig. 4). This would allow to use a relatively coarse mesh and still accurately describe septa slightly longer than an integer number of Δl.

The theory exposed for the description of the knife edge discontinuity is still valid; in particular the expression for the Green's function (3) remains the same, once we have centered the coordinate system (ρ, ϕ) on the corner of the septum. The field at the corner is modeled with a three-port equivalent circuit, since the metallic septum is placed on the nodes of the TLM mesh.

To determine the Z elements describing the circuit we must integrate the Green's function over a circular sector, for both the source points and the field points (Fig. 4). As an example, the expression for Z_{11} is shown in (14)

$$Z_{11} = \frac{1}{W_1^2}\int_0^{\phi_2} d\phi \int_0^{\phi_2} d\phi' \int_{\rho_1(\phi)}^{\rho_2(\phi)} \rho\, d\rho \cdot \int_{\rho_1'(\phi')}^{\rho_2'(\phi')} \overline{G}(\rho, \phi; \rho', \phi')\rho'\, d\rho'. \tag{14}$$

The evaluation of this kind of integral is particularly cumbersome, since the integrand is a series (3) and its argument is not uniquely defined over the domain of integration (depending on the ratio ρ/ρ'); moreover, since the center of the coordinate system is shifted with respect to the vertex of the circular sector, the domain of integration is not easily described. For these reasons, even a numerical evaluation of the integrals with an acceptable level of precision is burdensome.

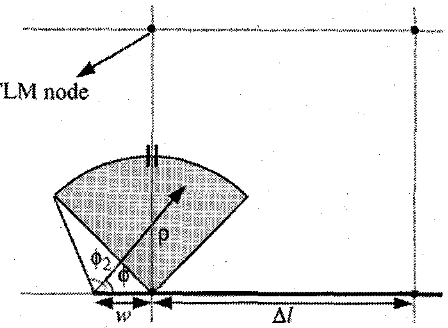


Fig. 4. Position of the knife edge septum of arbitrary length in the TLM mesh and domain of integration for the evaluation of Z_{ij}.

Consequently, an approximate solution for the values of the Z matrix has been found empirically. We have assumed a quadratic dependence and determined the coefficients experimentally comparing the obtained results with Marcuvitz's data. Defining with w the length of the septum extending beyond the TLM node (Fig. 4), the approximate expressions for the Z elements are given by

$$\begin{aligned} z_{11}(s) &= z_{11}(0)[-s^2 - s + 1] \\ z_{13}(s) &= z_{13}(0)[s^2 - 2s + 1] \\ z_{12}(s) &= z_{12}(0)[s^2 - 2s + 1] \\ z_{22}(s) &= z_{22}(0)[s^2 - 2s + 1] \\ s &= \frac{w}{\Delta l} > 0. \end{aligned} \tag{15}$$

In these formulas the values $z_{ij}(0)$ refer to the case previously analyzed, where the speptum length was equal to an integer number of Δl.

A similar set of expressions has been found also for the case of septa slightly shorter ($w < 0$) than an integer number of Δl

$$\begin{aligned} z_{11}(s) &= z_{11}(0)[-s^2 - 2s + 1] \\ z_{13}(s) &= z_{13}(0)[s^2 - s + 1] \\ z_{12}(s) &= z_{12}(0)[s^2 - s + 1] \\ z_{22}(s) &= z_{22}(0)[s^2 - \sqrt{2}s + 1] \\ s &= \frac{w}{\Delta l} < 0. \end{aligned} \tag{16}$$

These approximations give good results for values of $|s| \leq 0.35$. This restricted range does not represent a serious limitation; in fact, for most of the practical applications this condition can be easily verified with the choice of a relatively coarse mesh. On the other hand, (15) and (16) allow to treat perturbations of the septum length that are small, compared to the mesh size; these would otherwise require very fine discretizations to exactly describe the geometry.

VI. Results

The proposed method has been applied to analyze discontinuities in the transverse section of a rectangular waveguide

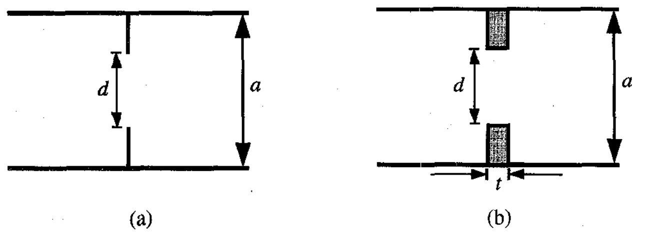


Fig. 5. Top view of the inductive irises in WR(28) waveguide. (a) Thin iris. (b) Thick iris.

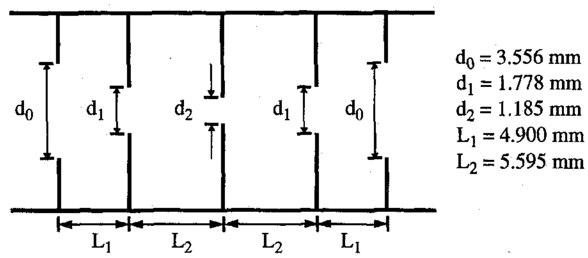


Fig. 7. Top view of the iris coupled bandpass filter in WR(28) waveguide.

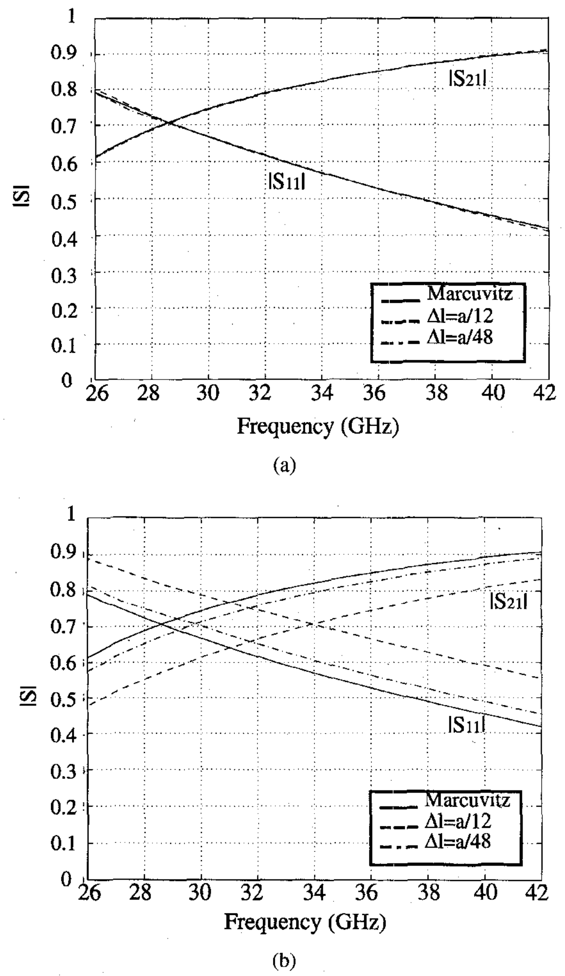


Fig. 6. S-parameters for the thin iris in WR(28) waveguide. (a) TLM with corner correction. (b) TLM without corner correction.

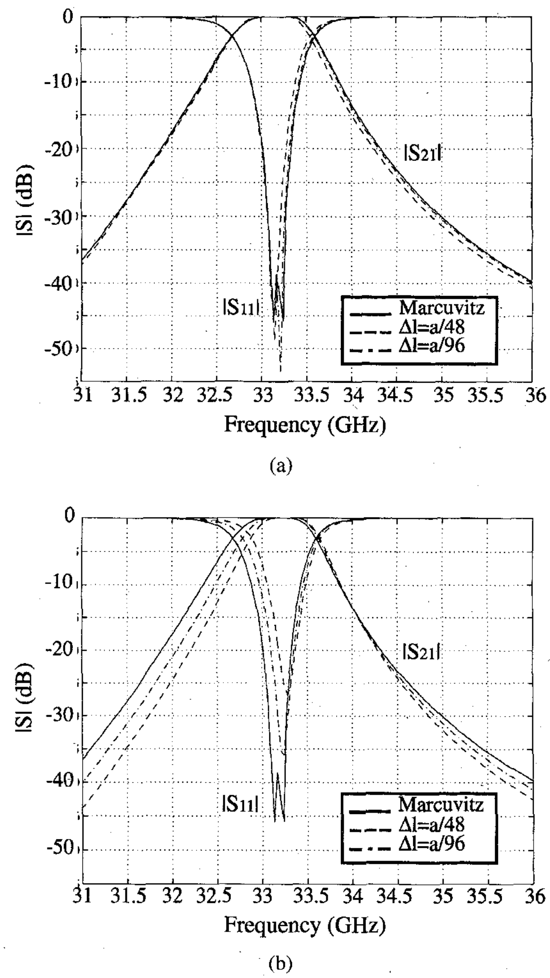


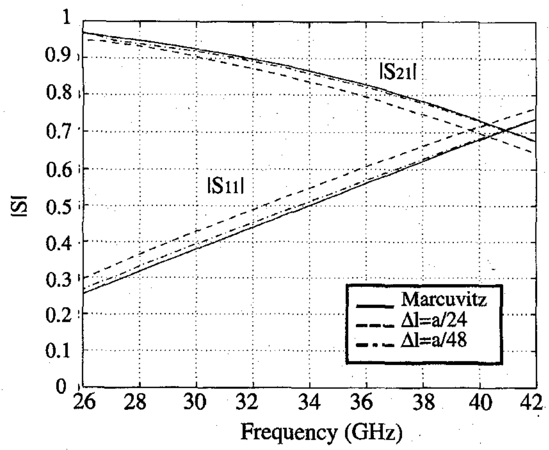
Fig. 8. Iris coupled bandpass filter in WR(28) waveguide. (a) TLM with corner correction. (b) TLM without corner correction.

(Fig. 5). To validate the model of a knife edge, a symmetrical inductive iris with aperture $d = 3.556$ mm $(a/2)$ in a WR(28) waveguide has been analyzed both with the corner modification and the regular TLM algorithm, and the results have been compared with Marcuvitz's [9] formulae. The scattering parameters obtained for different discretizations are shown in Fig. 6(a). Note that the corner modification improves considerably the accuracy of the TLM algorithm [Fig. 6(b)] even when a very coarse mesh is used.

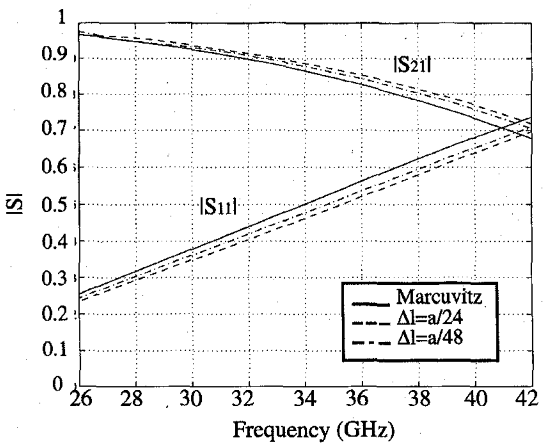
To further test the efficiency of the proposed method, an iris-coupled waveguide bandpass filter (Fig. 7), with center frequency of 33.18 GHz and bandwidth of 0.94 GHz, has been analyzed. Also in this case the corner correction results in a much faster convergence to Marcuvitz's curves as compared with the standard TLM algorithm [Fig. 8(a) and (b)].

To verify the model of the 90° wedge, a symmetrical iris, of thickness $t = 1.1853$ mm $(a/6)$ in a WR(28) waveguide has been examined. Comparison with the uncorrected TLM algorithm and other techniques has shown that in this case the correction is less effective since, for this kind of discontinuities, the standard TLM method provides good accuracy even with relatively coarse discretizations [Fig. 9(a) and (b)].

Finally, the correction for infinitely thin septa of arbitrary length in the TLM mesh has been validated, both for positive and negative variations of the length with respect to a integer number of Δl. A symmetrical inductive iris with aperture



(a)



(b)

Fig. 9. S-parameters for the thick iris in WR(28) waveguide. (a) TLM with corner correction, (b) TLM without corner correction.

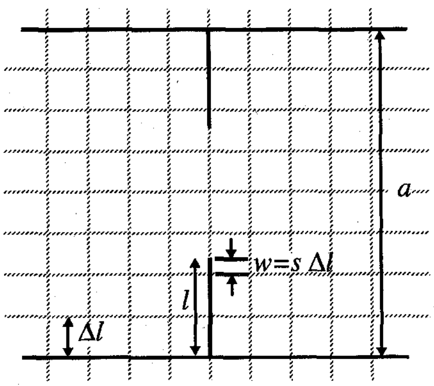
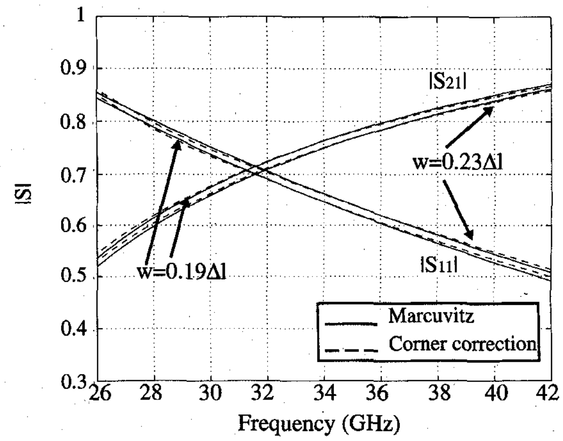
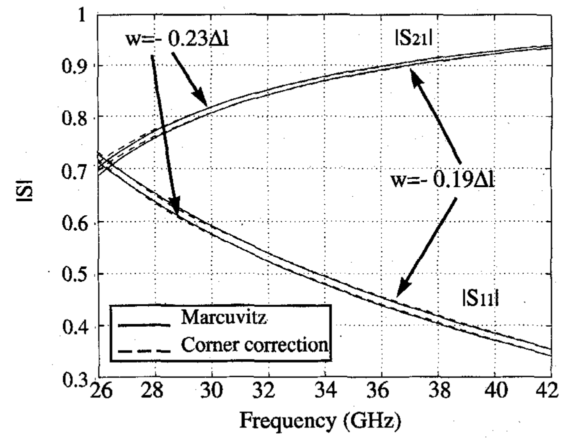


Fig. 10. Top view of the thin inductive iris in WR(28) waveguide with septa of arbitrary length l.

slightly different from $a/2$ has been analyzed with the corner modification (Fig. 10), and the results have been compared with Marcuvitz's formulae. In particular, we have discretized the waveguide width with $12\Delta l$, and considered the septum length $l = a/4 \pm s \cdot \Delta l$ (for $s = 0.19$ and 0.23). We have chosen lengths that, with the standard TLM method, would require a very fine mesh discretization to obtain an exact description.



(a)



(b)

Fig. 11. Parameters of a thin inductive iris in WR(28) waveguide for different lengths of the septa. (a) Discretization: $\Delta l = a/12$. $w = +0.19\Delta l, +0.23\Delta l$; (b) Discretization: $\Delta l = a/12$. $w = -0.19\Delta l, -0.23\Delta l$.

The scattering parameters obtained for the two structures are shown in Fig. 11(a) and (b). From the results we can see that the corner correction is always in good agreement with Marcuvitz's data, even if a coarse discretization is used. From these figures it is also possible to verify that the proposed technique is sensitive to small variations of the length; in fact, the curves obtained with the corner correction for a variation of $0.04\ \Delta l$ (from $s = \pm 0.19$ to $s = \pm 0.23$) are clearly distinct and in agreement with the theoretical results. To appreciate such a small difference in behavior by using the standard TLM method, we would need a 25 times finer discretization.

VII. Conclusion

In this paper we have derived an equivalent circuit for knife edges and 90° wedges, based on a quasi-static formulation of the field around the edge, and we have introduced it in the 2-D-TLM algorithm.

The proposed corner correction has been compared with the regular TLM method and with data available in the literature, and has yielded a noticeable improvement in the accuracy as well as in the convergence of the results for knife edges, while in the case of 90° wedges the standard TLM algorithm has proved to be sufficiently accurate.

The better description of the singular behavior of the field around the edge allows considerable savings in computer processing time and memory requirements when compared to mesh grading, since the desired accuracy can be achieved by using a coarser lattice.

An immediate extension of this method is its application to problems involving dielectric interfaces and sharp metallic boundaries, such as microstrips.

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Dr. Rozzi was awarded the Microwave Prize by the IEEE Microwave Theory and Technique Society in 1975. He is also a Fellow of the IEE (U.K.).

Wolfgang J. R. Hoefer, for a photograph and biography, see this issue, p. 2518.